COMBINED CONVECTION ON A VERTICAL PERMEABLE SURFACE FOR DIFFERENT PRANDTL NUMBERS

V. I. Dubovik

UDC 536.244

The combined action of free and forced convection occurs during heat exchange between a heated surface and a medium moving around it. In combined convection the laminar mode of motion is of most interest, when maximum velocity and temperature gradients are observed in the boundary layer around the surface. The effect of the buoyancy on the forced motion in laminar flow around a vertical impermeable plate was investigated in [1], where it was shown that the solution of the problem of combined convection can be represented in the form of a series in terms of the parameter $A = Gr/Re^2$, characterizing the ratio of the Grashof number to the Reynolds number.

Numerical solutions of combined-convection problems are presented in [2, 3] for an impermeable vertical surface.

In [2], using the method of local automodeling, in which the change in the velocity and temperature functions with respect to the variable A is neglected, combined convection is analyzed when the physical properties of the medium change as a function of the temperature. The automodeling problem of combined convection for uniform heat flows and wall temperature is considered in [3] for different values of A and an impermeable vertical surface. In [4] Soviet and foreign work on combined convection are generalized, and quantitative theoretical and experimental data on heat transfer and the resistance to the flow of an incompressible liquid over surfaces having different orientation are presented.

In this paper, we investigate numerically combined convection on a vertical permeable surface for forced and free flow in the same and opposite directions as a function of the Prandtl number.

The system of differential equations of the laminar boundary layer for combined convection which express the laws of conservation of mass, momentum, and energy, with the boundary conditions, are written in [5] on the assumption that the properties of the medium are constant, with the exception of the density, which depends on the temperature in the expression for the buoyancy, and ignoring viscous dissipation.

The equations in partial derivatives can be reduced to ordinary equations using a similar transformation to that employed in [5] for specified distributions of the surface temperature T_w and the velocity of unperturbed flow U_∞ on the external boundary of the boundary layer, specified by the relation

$$T_w - T_\infty = Bx^n, \quad U_\infty = Cx^m, \tag{1}$$

where C, B, m, and n are constants.

Taking into account Eqs. (1) and the condition $n = 2m - 1$, the system of differential equations can be reduced to the dimensionless equations

$$f'''(\eta) + (m+1) f(\eta) f''(\eta) - 2mf'^2(\eta) + 8\left[m \pm \frac{\mathrm{Gr}}{\mathrm{Re}^2}\theta(\eta)\right] = 0,$$
$$\theta''(\eta) + \mathrm{Pr}\,[(m+1) f(\eta)\theta'(\eta) - (4m-2) f'(\eta)\theta(\eta)] = 0, \tag{2}$$

where f is a dimensionless function of the velocity; $\theta = (T - T_\infty)/(T_w - T_\infty)$, dimensionless temperature; Gr, Re, and Pr, Grashof number, the Reynolds number, and the Prandtl number; and T_∞, temperature of the flow on the external side of the boundary layer.

In Eqs. (2) the primes indicate differentiation with respect to

$$\eta = (1/2)(C/\nu)^{1/2} x^{(m-1)/2} y,$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 129-132, January-February, 1981. Original article submitted August 6, 1979.

 0021-8944/81/2201-0108$07.50

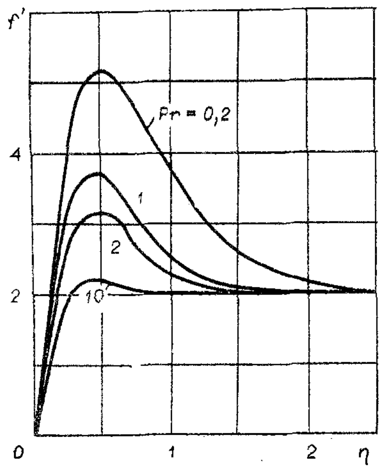

Fig. 1

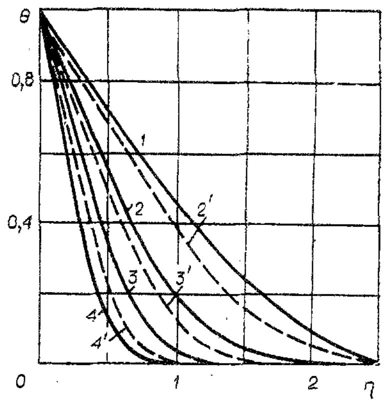

Fig. 2

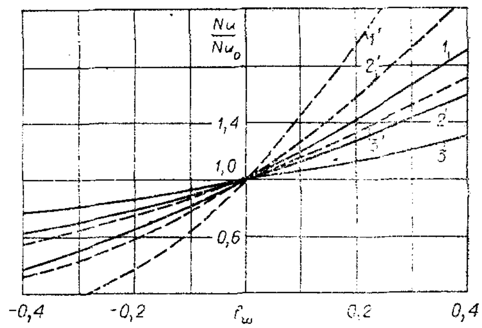

Fig. 3

where x, y, and ν are rectangular coordinates and the kinematic viscosity.

The boundary conditions in the new variables have the form

$$f'(0) = 0, \quad f_w = \text{const}, \quad \theta = 1 \quad \text{for} \quad \eta = 0,$$
$$f'(\infty) = 2, \quad \theta = 0 \quad \text{for} \quad \eta = \infty.$$

The condition f_w = const denotes that $v_w \sim x^{(m-1)/2}$. Using the component of the velocity v expressed in terms of the new variables, we obtain the blowing (suction) parameter

$$f_w = -[2v_w/(m+1)U_\infty]\sqrt{\mathrm{Re}};$$

the negative value of f_w corresponds to blowing ($v_w > 0$), and the positive value corresponds to suction ($v_w < 0$).

The system of equations (2) describing combined convection includes four parameters: Gr/Re^2, Pr, f_w, and m; when the flows are in the same directions we take the plus sign in the equations of motion, and when they are in opposite directions we take the minus sign.

Calculations were carried out on a computer using the method of iterations and the pivotal-condensation method [6] for $A = Gr/Re^2$, varying from 100 to −0.95, Pr = 0.2, 0.7, 1, 3, and 10, for m = 0.5 and n = 0, and for different values of f_w (positive and negative). The distributions of the dimensionless velocities and temperatures in the boundary layer are shown in Figs. 1 and 2. The dimensionless velocity profiles f'(η) in the boundary layer for combined convection A = 10 are shown in Fig. 1 for different values of the Prandtl criterion for f_w = −0.05. The buoyancy has its most important influence for motion at low Pr numbers. In Fig. 2, as an example, we show temperature profiles in the boundary layer for combined convection for the parameter A equal to 1 (the continuous curves) and −0.95 (the broken curves), for Pr = 0.2, 1, 3, and 10 (curves 1-4, respectively) for flows in the same direction, and Pr = 1, 3, and 10 (curves 2'-4') for the case of motion in opposite directions. In Fig. 2 it can be seen that the temperature profiles have a similar form both for positive and negative values of A, which is due to the fact that the nature of the variation in the

TABLE 1

$\frac{Gr}{Re^2}$	Pr 0,2	0,7	1	2	3	10	0,7[3]
			Values of $\theta'(0)$				
—0,95		0,5856	0,7014	0,9562	1,1384	1,8327	0,5984
—0,9		0,6117	0,7241	0,9771	1,1578	1,8507	0,6258
—0,8	0,5034	0,6564	0,7618	1,0137	1,1932	1,8853	0,6667
—0,5	0,5335	0,7384	0,8427	1,0989	1,2793	1,9709	0,7472
—0,1	0,5631	0,8166	0,9175	1,1827	1,3671	2,0682	0,8179
0	0,5695	0,8293	0,9326	1,2013	1,3853	2,0883	0,8323
0,1	0,5754	0,8422	0,9521	1,2177	1,4084	2,1072	—
1	0,6204	0,9326	1,0553	1,3412	1,5414	2,2634	0,9425
10	0,8341	1,3229	1,4971	1,8806	2,1433	3,0533	1,3466
100	1,3728	2,2059	2,4944	3,1104	3,5391	4,9415	2,2698
			Values of $f''(0)$				
—0,95		0,0028	0,2763	0,8053	1,0845	1,7848	0,0033
—0,9		0,2812	0,5179	0,9906	1,2439	1,8719	0,2953
—0,8	0,3959	0,7684	0,9421	1,3365	1,5466	2,0798	0,7904
—0,5	1,7165	1,9472	2,0475	2,2518	2,3661	2,8885	1,9852
—0,1	3,2124	3,2576	3,2754	3,3116	3,3319	3,3867	3,2980
0	3,5631	3,5631	3,5631	3,5631	3,5631	3,5631	3,5989
0,1	3,8951	3,8541	3,8372	3,7994	3,7835	3,7266	—
1	6,6603	6,2349	6,0854	5,7855	5,6138	5,1463	6,2804
10	26,317	23,072	22,044	20,064	18,909	15,796	23,187
100	140,455	121,371	115,551	104,430	97,959	80,767	122,59

boundary layer is the same for free and forced convection. The calculated values of f"(0) and θ'(0) for different values of A and Pr for an impermeable surface are given in Table 1; here we present the results of calculations for flows in the same and opposite directions with Pr = 0.7 taken from [3]. The agreement with the data obtained from [3] is quite satisfactory. For heat exchange with combined convection we have [5]

$$Nu = -0.5\, Re^{0.5}\theta'(0), \quad (3)$$

where Nu is the Nusselt number.

The quantities θ'(0) for the calculation using Eq. (3) were calculated on a computer for different values of the parameters Gr/Re^2, Pr, and f_w. As an example, the results of calculations of the heat exchange Nu/Nu_0 for combined convection on a permeable surface are given in Fig. 3, where curves 1-3 correspond to the values Gr/Re^2 = 0, 10, and 100, for Pr = 3, while curves 1'-3' are for Pr = 10. In Fig. 3 the Nusselt number Nu_0 was taken for an impermeable surface.

Analysis of the results of the numerical calculation shows that the most important effect of the buoyancy on the motion for combined convection occurs for low Prandtl numbers when the flows are in the same direction. When the flows are in opposite directions the positive pressure gradient of free convection causes a slowing down of the velocities and an increase in the thickness of the boundary layer. The flow of heat and the friction stress increase when the flows are in the same direction and decrease when the flows are in opposite directions.

LITERATURE CITED

1. E. M. Sparrow and J. L. Gregg, "Buoyancy effects in forced-convection flow and heat transfer," Trans. ASME, Ser. E, J. Appl. Mech., 26, No. 1, March (1959).
2. S. Tsuruno and S. Nagai, "Buoyancy effects in forced-convection flow and heat transfer," Bull. JSME, 12, No. 53 (1969).
3. E. M. Sparrow, R. Eichhorn, and J. L. Gregg, "Combined forced and free convection in a boundary layer flow," Phys. Fluids, 2, No. 3 (1959).
4. O. G. Martynenko and Yu. A. Sokovishin, Heat Transfer by Mixed Convection [in Russian], Nauka i Tekhnika, Minsk (1975).
5. V. I. Dubovik, "Friction and heat exchange in combined convection on a permeable surface," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1973).
6. P. M. Brdlik et al., "Laminar free convection on a vertical surface complicated by condensation or evaporation," Nauchn. Trud. Nauchn.-Issled. Inst. Stroit. Fiz., Gosstroya SSSR, No. 2 (1967).